INVESTIGATING REYNOLDS SCALING OF TIP-LEAKAGE VORTEX FLOW VIA NUMERICAL SIMULATIONS

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Abstract To study the Reynolds number scaling of cavitation inception, a ducted propulsor simulation is developed and extensively validated with experimental results. The simulation is used as a tool for investigating the minimum pressure, circulation, and axial/tangential velocities in the vortex core, thus providing a means of correlating these quantities to Reynolds number. The simulation reveals that the leakage vortex exhibits little dependence on Reynolds number, while the trailing edge vortex appears to exhibit classical trends. Moreover, the trailing edge, albeit the weaker vortex, appears to be causing inception.

Keywords: cavitation, inception, tip, leakage, flow, vortex, unstructured, computational fluid dynamics

1. Introduction

Tip-leakage vortex cavitation inception is the primary cause of cavitation inception for ducted propulsors. Little has been published on the tip-leakage vortex interaction from an experimental approach. One notable exception is a paper by Farrell and Billet (Farrell and Billet, 1994), who develop a correlation to predict leakage vortex minimum pressure. There is even less work on computational simulation of tip-leakage flows. Another notable exception is a paper by Lee et al. (Lee et al., 1998), who numerically model the leakage vortex flow evident in ducted propulsors using a structured approach. In this work, an unstructured simulation is developed to study Reynolds number scaling of tip-leakage vortex cavitation inception, and is discussed in more detail in (Brewer, 2002). Simulations were performed at a variety of Reynolds numbers ranging from one million to one billion, representative of model to full scale Reynolds numbers.

2. Simulation methodology

Recently, Mississippi State University developed an unstructured, Reynolds-Averaged Navier-Stokes (RANS) solver, named U^2NCLE. The unstructured approach allows for automated nodalization of complex geometries using the Advancing-Front/Local-Reconnection (AFLR) unstructured grid generator (Marcum, 1995). U^2NCLE solves the finite-volume based discretization of the integral form of the RANS equations (Hyams, 2000):

$$\frac{\partial}{\partial t}\int_{\Omega} Q d\mathcal{V} + \int_{\partial\Omega} \vec{F}\cdot\vec{n} dA = \frac{1}{Re}\int_{\partial\Omega} \vec{G}\cdot\vec{n} dA + \int_{\Omega} s d\mathcal{V} \qquad (1)$$

where Re is the Reynolds number, $Q = [p, u, v, w]^T$, $\vec{F}$ is the inviscid flux vector, $\vec{G}$ is a viscous flux vector containing the shear stress terms, and s is a source term to account for relative motion of the propeller. Equation 1 is discretized over small vertex-centered control volumes Ω created by the median dual of the surrounding vertices. This equation is solved interactively with a one-equation Spalart-Allmaras turbulence model. The system of equations are solved using a discretized Newton relaxation (DNR) scheme, as discussed in (Whitfield and Taylor, 1991), where the system of equations are solved using a symmetric Gauss-Seidel relaxation scheme.

The simulation is performed on a ducted propulsor which was also experimentally investigated in the 36-inch water tunnel at the NAVSEA Surface Warfare Center, Carderock Division. Figure 1 shows the resulting trailing vortex structure from both the experiment and the simulation.

A numerical post-processing analysis tool was developed to quantitatively investigate the resulting vortex structure. The method interrogates the simu-

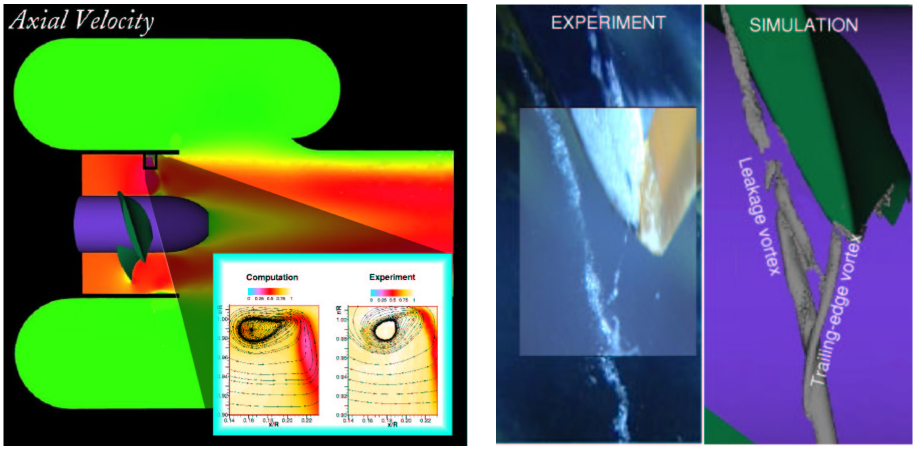

Figure 1. Simulation of ducted propulsor in 36-inch water tunnel test section. *Left:* axial and streamwise velocity distribution. *Right:* Cavitation comparisons with experiment.

Table 1. Reynolds dependence on $-C_{P_{min}}$, γ, v_t^{avg}, and v_s.

Re	γ	$-C_{P_{min}}$	v_t^{avg}	v_s
1.0×10^6	0.0710	4.8110	1.2496	1.6619
2.0×10^6	0.0813	5.0051	1.3231	1.6875
1.2×10^7	–	5.3166	1.3644	1.7095
1.0×10^8	–	5.8937	1.5021	1.7802
1.0×10^9	–	6.0776	1.5181	1.8217

lated flowfield to extract such quantities as: (1) the average vortex diameter a, (2) circulation γ, (3) average v_t^{avg} and maximum v_t^{max} tangential velocities, (4) minimum pressure $-C_{P_{min}}$, and (5) the vortex core streamwise velocity v_s.

3. Scaling analysis

As shown in Figure 1, the leakage vortex is the predominant vortex, which is easier to compute and analyze. The simulation was interrogated along the leakage vortex core axis to find minimum core pressure, maximum average tangential velocity, and streamwise core velocity. The Reynolds dependence of these variables are given in Table 1. From these results, the following trends are observed for the leakage vortex:

$$-C_{P_{min}} \sim Re^{0.03} \qquad v_t^{avg} \sim Re^{0.03} \qquad v_s \sim Re^{0.01} \qquad \gamma \sim Re^{0.02} \tag{2}$$

Thus, the leakage vortex is shown to exhibit a very low dependence on Reynolds number. A similar trend in Reynolds number dependence of the leakage vortex is shown in (Green, 1989). However, the trailing edge vortex exhibit more classical trends that are evidenced in previous experimental investigations.

The scaling exponent, n, is computed from the computational results in the following manner:

$$n = \log_b \left(\frac{C^s_{P_{min}}}{C^m_{P_{min}}} \right) \quad \text{where} \quad b = \frac{Re_s}{Re_m} \tag{3}$$

In this analysis, Re_m was taken constant at 1.0×10^6, while Re_s was allowed to vary through the Reynolds number range. Expressing this relationship in the form of a power law gives:

$$\frac{\sigma_s}{\sigma_m} = \frac{C^s_{P_{min}}}{C^m_{P_{min}}} = \left(\frac{Re_s}{Re_m} \right)^n \tag{4}$$

The following table shows the computed results of the comparison between the leakage vortex scaling exponent n_{LV} and the trailing-edge vortex exponent n_{TEV} for different model to full-scale scaling ratios Re_s/Re_m.

Re_s/Re_m	n_{LV}	n_{TEV}
2	0.057	0.350
12	0.051	0.323
100	0.044	0.306
1000	0.034	0.264

4. Summary

A Reynolds number scaling analysis using an unstructured RANS simulation revealed that the leakage vortex has a low dependence on Reynolds number, while the trailing-edge vortex exhibits a traditional scaling relationship. However, it is the interaction of both the leakage and trailing-edge vortex that causes cavitation to incept where the two vortices coalesce.

Acknowledgments

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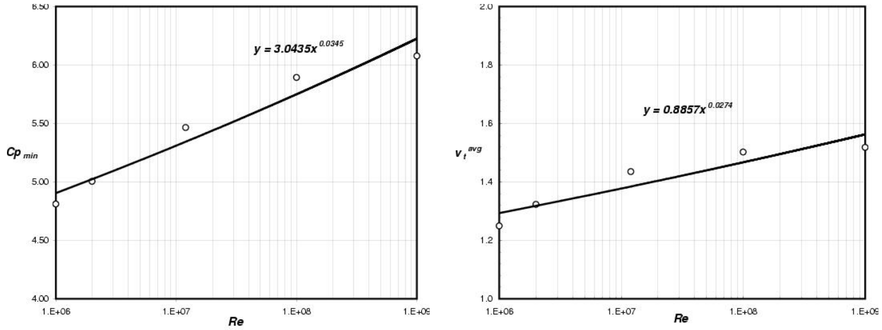
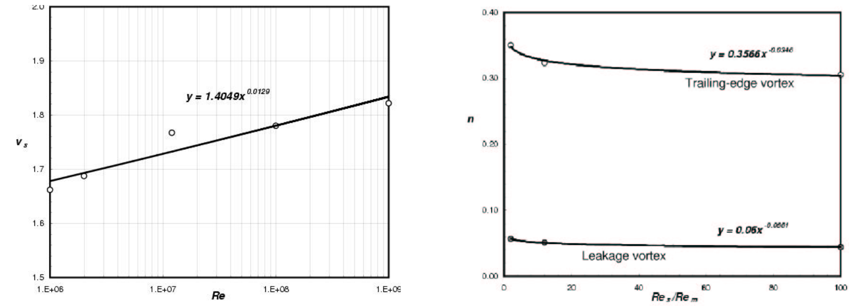
References

Brewer, W. H. (2002). *On simulating tip-leakage vortex flow to study the nature of cavitation inception.* Ph.D. dissertation, Mississippi State University, May 2002.



a. minimum core pressure

b. average tangential velocity



c. streamwise core velocity

d. functional relationship of n

Figure 2. Reynolds number dependence on pressure, velocity, and scaling exponent.

Farrell, K. J., and M. L. Billet (1994). “A Correlation of Leakage Vortex Cavitation in Axial-Flow Pumps,” Journal of Fluids Engineering [September].

Green, S. I. (1998). “Trailing Vortex Core Unsteadiness - an Exploratory Study of Reynolds Number Effects,” AIAA Journal, 88-3623-CP.

Hyams, Daniel (2000), “An Investigation of Parallel Implicit Solution Algorithms for Incompressible FLows on Unstructured Topologies,” Ph.D. Dissertation, Mississippi State University [May].

Lee, Y.-T., J. Feng, C. Merkle, and M. Tse (1998). “Effects of Tip-Clearance Flows,” Proceedings of the Twenty-Second Symposium on Naval Hydrodynamics, pp. 959-972, Washington, D.C.,

Marcum, D. L., and N. P. Weatherhill (1995). “Unstructured Grid Generation Using Iterative Point Insertion and Local Reconnection.” AIAA, 33(9) [September], pp. 1619-1625.

Shen, Y.T., Stuart D. Jessup, and Scott Gowing (2001). “Tip Vortex Cavitation Inception Logarithmic scaling.” Naval Surface Warfare Center Technical Report, NSWCCD-50-TR–2001/060 [October].

Whitfield, D. L. and L. K. Taylor (1991). “Discretized Newton-Relaxation Solution of High Resolution Flux-Difference Split Schemes,” AIAA Paper No. 91-1539 [June].